PHYSICAL REVIEW E **68**, 065102(R) (2003)

From multiplicative noise to directed percolation in wetting transitions

F. Ginelli,[1,2] V. Ahlers,[3] R. Livi,[4,2] D. Mukamel,[5] A. Pikovsky,[3] A. Politi,[1,2] and A. Torcini[1,2]
[1]*Istituto Nazionale di Ottica Applicata, Largo E. Fermi 6, I-50125 Firenze, Italy*
[2]*Istituto Nazionale di Fisica della Materia, Unità di Firenze, Italy*
[3]*Department of Physics, University of Potsdam, Potsdam, Germany*
[4]*Dipartimento di Fisica, Università di Firenze, Italy*
[5]*Department of Physics of Complex Systems, The Weizmann Institute, Rehovot, Israel*
(Received 4 March 2003; published 11 December 2003)

A simple one-dimensional microscopic model of the depinning transition of an interface from an attractive hard wall is introduced and investigated. Upon varying a control parameter, the critical behavior observed along the transition line changes from a directed-percolation type to a multiplicative-noise type. Numerical simulations allow for a quantitative study of the multicritical point separating the two regions. Mean-field arguments and the mapping on yet a simpler model provide some further insight on the overall scenario.

DOI: 10.1103/PhysRevE.68.065102 PACS number(s): 05.70.Ln, 64.60.−i, 05.45.Xt

A variety of interesting physical phenomena corresponds to the unbinding transition of an interface from a flat surface. This is the case of wetting processes (WP) taking place in the thin liquid film which forms on a substrate exposed to a gas. By varying external parameters such as temperature or pressure, the liquid layer $h(x,t)$ may either grow and become macroscopically thick or remain confined to the close vicinity of the substrate [1,2]. Wetting phenomena can also take place under nonequilibrium conditions. Here one is interested, for example, in a growth process of a film over a substrate. Depending on the dynamical rates controlling the growth process one can observe similar pinned or unpinned phases.

A question of general interest concerns the universality of the unbinding transition. While the equilibrium scenario is well established [1], an overall understanding of nonequilibrium wetting phenomena is still lacking. Numerical studies of nonequilibrium systems have revealed a composite picture that still needs to be fully disentangled. Here one considers the dynamical equations of a moving interface interacting with a hard wall. In the simplest case, the unbinding transition is signalled by the change of sign of the average velocity of the free interface. A pinned phase is obtained when the free interface moves towards the substrate and an unpinned phase is found when it moves away. This scenario is sometimes referred to as the multiplicative noise (MN) and is well described by a Kardar-Parisi-Zhang (KPZ) equation [3] with a hard wall [4–6]. On the other hand, a study of some discrete growth models has shown that for particular values of the growth rates the unbinding transition is of different nature, belonging to the directed-percolation universality class [7].

More interesting is the scenario when the surface exerts, in addition, a short-range attractive force that may prevent h from growing even when the free interface would otherwise have a positive velocity. In WP such an effective interaction is obtained when the growth rate on the bare substrate is lower than the growth rate on the film itself. Some microscopic models, introduced to study this out-of-equilibrium depinning transition [8,9], indicate that for a sufficiently strong attractive force, the unbinding transition may become discontinuous.

This scenario is by no means restricted to WP. Similar features have been, e.g., observed at the onset of complete synchronization in chains of coupled maps [10], where two replicas of the system are either affected by the same stochastic force or locally coupled. In this context, synchronization is quantified by the absolute difference $\Delta(i,t)$ between the state variables in the ith site and it can be easily recognized that $\ell(i,t)=-\ln(\Delta(i,t)/\Delta_{max})$ plays the same role as $h(x,t)$. Indeed, the synchronized regime, characterized by an exponentially decaying Δ is equivalent to the unpinned phase, with a linearly growing ℓ. On the other hand, the unsynchronized regime, characterized by a stationary distribution of Δ values, corresponds to the pinned phase with a wall in $\ell=0$ preventing Δ to be larger than the maximum possible value Δ_{max}. Finally, the amplification of small but finite perturbations that may be induced by strong and localized (in phase-space) nonlinearities [11] (e.g., discontinuities of the local map) is equivalent to an attractive force that drives the interface towards the wall in WP. However, at variance with WP, in complete synchronization, the effect of a sufficiently strong ''attractive force'' is to bring the MN transition into the directed-percolation (DP) universality class [10,12] rather than making it first order.

In the absence of a sufficiently general field-theoretic approach, able to reconcile all the various observed scenarios into a common framework, the study of minimal models is very helpful for the identification of the basic mechanisms. This is the main motivation for introducing hereafter a simple microscopic model. By numerically reconstructing its phase diagram, we shall be able to clearly recognize that both the MN and DP universality classes can be found and to quantitatively investigate the ''multicritical'' point separating the two scenarios. Moreover, an accurate reconstruction of the critical line will be proposed based partly on mean-field arguments and exploiting the exact mapping (in a limit case) onto a genuine DP model.

Specifically, we consider a simple growth model, which, for reasons that will become clear in a while, is called *single-step-plus-wall* (SSW) model. The starting point is the single-step (SS) model originally introduced to study the roughening of one-dimensional (1d) interfaces [13,14]. It is well

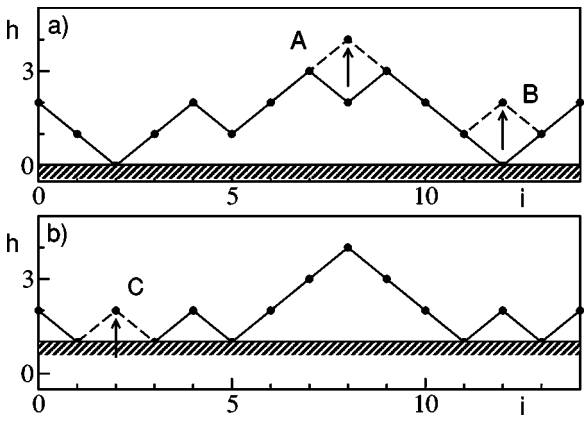

FIG. 1. Updating rule of the SSW model. The full line represents the interface, while the shaded area identifies the wall. Dashed segments indicate interface flips occurring in randomly chosen local minima [see *A* and *B* in panel (a)] and in all sites located below the wall after it has been shifted upwards by one unit [see *C* in panel (b)].

known that the SS model can be exactly mapped onto the 1d KPZ equation [13]. The interface is described by a set of integer heights h_i on the sites i of a one-dimensional lattice of length L, satisfying the "continuity" constraint $|h_i - h_{i+1}| = 1$. At each time step, $dt = 1/L$ a site i is randomly selected and its height increased, $h_i \to h_i + 2$, provided that a local minimum exists at site i. In the thermodynamic limit, since the dynamics does not introduce any spatial correlation, a generic interface with mean slope s moves with a mean velocity $v(s) = (1 - s^2)/2$. The exact knowledge of $v(s)$ will be crucial in the following, since it allows for an exact determination of the critical line in the MN regime.

The second ingredient of the SSW model is an upward-moving wall located at some integer height $h_w(t)$ below the SS interface. It moves with velocity v_w and both "pushes" and attracts the interface. Altogether, the SSW dynamics amounts to the following evolution algorithm: at each time step, a site i is randomly chosen and, if it is a local minimum, h_i is increased by two units with probability 1 or $(1-q)$, depending whether $h_i > h_w$ or $h_i = h_w$ [see Fig. 1(a)]. After $n_w = L/v_w$ steps, the wall is moved upwards by one unit and, simultaneously, the height of the interfacial sites overtaken by the wall is increased by two units (see Fig. 1b). Physically, the SSW describes a roughening and moving interface, attracted by a short-range force to a hard wall. Its dynamics is determined by two parameters: (i) the relative velocity of the wall with respect to the free-interface, which we control by modifying v_w [15] and (ii) the stickiness of the wall, quantified by q. Since we are interested in characterizing the phase diagram of SSW by locating the depinning transition from the wall, the natural order parameter is the density of sites pinned at the wall

$$\rho(t) = \frac{2}{L}\left\langle \sum_{i=1}^{L} s_i(t) \right\rangle, \quad s_i(t) = \begin{cases} 1 & \text{if} \quad h_i(t) = h_w \\ 0 & \text{if} \quad h_i(t) > h_w, \end{cases} \tag{1}$$

where $\langle \cdot \rangle$ denotes an ensemble average over different realizations of the stochastic process.

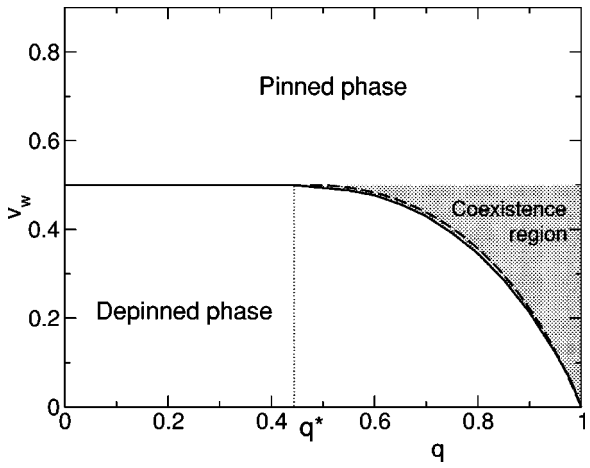

FIG. 2. Phase diagram of the SSW model. The depinning transition takes place along the solid line; the dashed line is the result of the analytic approximation discussed in the text. In the shaded area, stationary pinned interfaces exist even though v_w is smaller than the free-interface velocity $(=1/2)$.

A necessary condition for interface depinning to occur is $v_w < 1/2$, because 1/2 is the velocity of a free and flat interfaces. For q small enough, this is also a sufficient condition, since the attractive force is overcompensated by the faster velocity of an almost straight interface. This can be seen by a simple mean-field argument: an interface completely attached to the wall has a higher density of minima (1/2) than a rough one (1/4), so that its average velocity $v = (1-q)$ is larger than 1/2 as long as $q < q^* = 1/2$. Accordingly, below q^*, a pinned interface detaches as soon as $v_w < 1/2$. Numerical simulations confirm that the transition indeed occurs at $v_w = 1/2$, with the only slight difference that $q^* = 0.4445(5)$. Above q^* the interface may remain pinned even when its velocity is larger than v_w (see Fig. 2). In other words there is a sort of bistable region, where an initially pinned interface remains attached while a depinned one moves away from the wall. The transition line, located at the lowest border of the bistable region, is continuous and turns out to belong to the DP universality class. This is at variance with the discontinuous transition observed, e.g., in the solid-on-solid model of Ref. [9]. Before commenting on the possible reason of such a difference, it is necessary to explore in more quantitative way the critical behavior both above and below q^*.

Continuous nonequilibrium phase transitions are characterized by three independent critical exponents. At criticality $(v_w = v_w^c)$, the density $\rho(t)$ of pinned sites scales with time as $\rho \sim t^{-\delta}$, while its stationary value depends on the the distance from criticality as,

$$\lim_{t\to\infty} \rho(t) \sim (v_w - v_w^c)^{\beta}. \tag{2}$$

Very accurate numerical estimates of the DP critical exponents give $\beta_{DP} = 0.276\,486 \pm 6 \times 10^{-6}$ and $\delta_{DP} = 0.159\,464 \pm 6 \times 10^{-6}$ [16]. Less accurate estimates are available for the MN scenario, namely, $\beta_{MN} = 1.7 \pm 0.1$ and $\delta_{MN} = 1.1 \pm 0.1$ [5]. The third exponent, z, can be defined with reference to

TABLE I. Critical exponents of the SSW model. In parentheses we report the estimated uncertainty on the last figure.

q	v_w^c	δ	β	z
0	0.5	1.14(5)	1.70(5)	1.5(1)
0.2	0.5	1.13(5)	1.75(5)	1.5(1)
0.4	0.5	1.12(5)	1.5(1)	1.3(1)
0.4445(5)	0.5	0.50(1)	0.74(5)	1.5(1)
0.6	0.476 35(5)	0.15(1)	0.276(5)	1.5(1)
0.7	0.429 75(5)	0.16(1)	0.27(1)	1.5(1)
0.8	0.348 95(5)	0.17(1)	0.276(5)	1.5(1)

the scaling relation $\rho(t,L)=L^{-\delta z}g(tL^{-z})$ [2], where g is a proper scaling function. It takes very similar values in DP and MN, namely, $z_{DP}=1.580\,745\pm10^{-6}$ while $z_{MN}=1.53\pm0.07$.

We have performed numerical simulations of the SSW model for different q values starting from an initially pinned $(\rho(0)=1)$ interface. The exponents β and δ have been measured by studying large system sizes (from $L=2^{17}$ to $L=2^{20}$) in order to minimize finite-size corrections, and by averaging over a small number of different realizations ($\approx$10) to further decrease statistical fluctuations. Conversely, it is sufficient to consider much smaller sizes ($2^5<L<2^{10}$) for a reliable estimate of z that has been determined by looking for the optimal collapse of the various $\rho(t,L)$ curves [in this case, however, it has been necessary to average over a much larger ensemble of realizations ($>10^4$–10^5)].

A complete summary of the resulting values of the critical exponents is reported in Table I. Altogether, we find that for $q<q^*$, the attracting force not only does leave the transition point unaffected but also the universality class of the critical behavior remains of MN type [17]. For $q>q^*$, the transition line veers down, while the critical exponents signal a transition of DP type.

The DP critical line can be best understood by analyzing the SSW model in the vicinity of the point $q=1$ and $v_w=0$. Here, the dynamics is dominated by two slow mechanisms: (i) detachment of pinned sites during the asynchronous part of the rule and (ii) shrinking of the unpinned islands at the wall move. In comparison, the dynamics of detached regions between consecutive wall moves rapidly leads them to assume a perfectly triangular shape with a maximal slope equal to ±1. Therefore, such islands correspond to the dead phase in DP, since their shape prevents the occurrence of any pinning in their interior. It is now convenient to divide between attached ($h_i=h_w$) and detached ($h_i>h_w$) sites, denoting them with A and D, respectively. Correspondingly, the dynamics reduces to a simple probabilistic cellular automaton: L/v_w sites are first randomly selected, transforming each A into a D with probability $1-q$; next, the wall move amounts to transforming all A's into D's and all D's neighboring an A into an A (see Fig. 3). Apart from the peculiar presence of both an asynchronous and a synchronous part, this rule clearly belongs to the class of contact processes with an absorbing state (D) and, as such, it is expected to exhibit a DP transition [18,19]. The only relevant

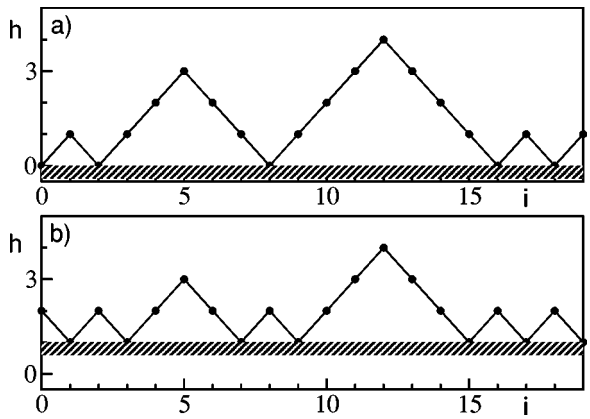

FIG. 3. The interface before [panel (a)] and after [(b)] the wall move, with the same settings as in Fig. 1. The corresponding configurations read as *ADADDDDDADDDDDDDADAD* and *DADADDDADADDDDDADADA* (see text for the definition of *A* and *D*.)

parameter is the ratio between the rate $(1-q)$ and the wall velocity v_w, which corresponds to the slope of the critical line at $q=1$. In fact, numerical simulations of the automaton yield a "critical ratio" $a_c=\lim_{q\to0}v_w^c/(1-q)=2.866\ldots$, in very good agreement with the slope determined from direct simulations of the SSW model.

At a finite distance from $q=1$, the automaton does no longer describe exactly the SSW dynamics, since fluctuations of the interface within an unpinned island can both induce a faster shrinking of the island and the generation of pinned sites in its interior. The first effect amounts to increasing by some factor of α the average number of D's turned into A's at the island borders, so that the critical point would be determined by the equation

$$\frac{\alpha v_w^c}{1-q}=a_c. \tag{3}$$

In the configuration plotted in Fig. 1, the island border identified by C shifts by three units, thus implying $\alpha=3$. The average value of α can be determined by noticing that, at criticality, in the SSW model, the typical slope s of the interface inside an island must be such that its velocity coincides with the wall velocity v_w^c. The value of α can be determined by assuming that the profile is a biased random walk with probabilities p_u and $1-p_u$ of up and down moves, respectively, and neglecting the contribution due to the onset of isolated pinnings inside the islands. Under these hypotheses, α coincides with the average number of steps made by one such biased walker (starting at the wall height h_w) before it reaches $h=h_w+2$ for the first time, i.e., $\alpha=1+2(1-p_u)/p_u$. Since p_u is simply related to the slope s by $p_u=(s+1)/2$, one finally obtains $\alpha=1+2(1-s)/(1+s)$. Inserting this expression into Eq. (3) and eliminating s with the help of the relation $v_w^c=(1-s^2)/2$, one obtains $v_w^c(q)=2\sqrt{1+2a_c(1-q)}-a_c(1-q)-2$. Although approximate, this formula reproduces very accurately the DP critical line not only in the vicinity of $q=1$ but also up to the multicriti-

cal point, where it touches the MN critical line (see Fig. 2), providing a good approximation for its position as well [$q_{\max}=1-3/(2a_c)=0.477\ldots$]. Evidently, the quality of the theoretical formula implies that even close to the multicritical point, the sudden appearance of pinned sites inside unpinned islands does not significantly modify the transition point. A deeper understanding of this point is left to future investigations.

The correspondence with MN and DP critical phenomena unveiled for small and large q values, respectively, is not sufficient to make predictions about the scaling behavior in the vicinity of the multicritical point [20]. There, in the absence of a convincing field-theoretic approach, a chance for understanding how the two out-of-equilibrium critical phenomena may be connected to one another is offered by numerical investigation. However, even this is not a straightforward task, since three levels of criticality mix together: criticality of the free rough interface, criticality of the depinning transition, and, finally, that one connected with the MN-DP transition. While approaching q^* from the left along the critical line $v_w^c=1/2$, the power-law decay of the density of pinned sites $\rho(t)$ turns out to be first governed by the exponent $\delta^*\approx 1/2$, which crosses over to δ_{MN}. As the crossover time appears to diverge when $q\to q^*$, it can be safely stated that δ^* characterizes the critical behavior at q^*. The same scaling is expected for q approaching q^* from the right along the critical line, with a crossover from δ^* to δ_{DP}. However, the difficulty of locating the critical line with a sufficient accuracy prevents an effective numerical verification. The other two critical exponents are $\beta^*\approx 3/4$ and $z^*\approx 3/2$ at q^*, so that both the β^* and the δ^* values are intermediate between the corresponding MN and DP critical exponents, while z^* is compatible with both z_{MN} and z_{DP}.

In conclusion, with reference to a simple microscopic model, we have shown that MN and DP can be different facets of the same wetting process. The connection between these two different universality classes strongly hints at the possibility that both out-of-equilibrium transitions may be described within a single field-theoretic approach. Some progress has been recently made in this direction in Ref. [21], where the authors have found DP behavior in a KPZ equation with attractive wall.

We are grateful to P. Grassberger for his contributions in the early stages of this work. A. Giacometti and L. Giada are acknowledged for useful discussions. CINECA in Bologna provided us access to the parallel Cray T3 computer under the INFM-grant ''Iniziativa Calcolo Parallelo.'' This work has been partially funded by the FIRB-Contract No. RBNE01CW3M_001 and the Israeli Science Foundation.

[1] S. Dietrich, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J.L. Lebowitz (Academic Press, London, 1988), Vol. 12, p. 1.
[2] H. Hinrichsen, Adv. Phys. **49**, 815 (2000).
[3] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (1986).
[4] G. Grinstein, M.A. Muñoz, and Y. Tu, Phys. Rev. Lett. **76**, 4376 (1996).
[5] Y. Tu, G. Grinstein, and M.A. Muñoz, Phys. Rev. Lett. **78**, 274 (1997).
[6] M.A. Muñoz and T. Hwa, Europhys. Lett. **41**, 147 (1998).
[7] U. Alon, M.R. Evans, H. Hinrichsen, and D. Mukamel, Phys. Rev. Lett. **76**, 2746 (1996).
[8] H. Hinrichsen, R. Livi, D. Mukamel, and A. Politi, Phys. Rev. Lett. **79**, 2710 (1997).
[9] H. Hinrichsen, R. Livi, D. Mukamel, and A. Politi, Phys. Rev. E **61**, R1032 (2000).
[10] L. Baroni, R. Livi, and A. Torcini, Phys. Rev. E **63**, 036226 (2001); V. Ahlers and A. Pikovsky, Phys. Rev. Lett. **88**, 254101 (2002).
[11] A. Politi and A. Torcini, Europhys. Lett. **28**, 545 (1994).
[12] F. Ginelli, R. Livi, A. Politi, and A. Torcini, Phys. Rev. E **67**, 046217 (2003).
[13] P. Meakin, P. Ramanlal, L.M. Sander, and R.C. Ball, Phys. Rev. A **34**, 5091 (1986).
[14] J. Krug and H. Spohn, in *Solids Far From Equilibrium*, edited by C. Godréche (Cambridge University Press, Cambridge 1991), p. 479.
[15] The reason for this choice is purely technical: the nonlinear term of the KPZ equation describing the evolution of the free interface is proportional to its velocity [13]. Since this term is also responsible for the out-of-equilibrium dynamics, we prefer to keep it constant.
[16] I. Jensen J. Phys. A **32**, 5233 (1999).
[17] For $q=0.4$, the estimate of β is not in good agreement with the known MN value. This can be probably ascribed to finite-size effects due to the proximity of the multicritical point q^*.
[18] H.K. Janssen, Z. Phys. B Condens. Mater **42**, 151 (1981).
[19] P. Grassberger, in *Nonlinearities in Complex Systems*, edited by S. Puri and S. Dattagupta (Narosa Publishing House, New Delhi, 1997).
[20] The existence of a multicritical point has been conjectured also in the context of the synchronization transition; see A. Lipowski and M. Droz, Phys. Rev. E **67**, 056204 (2003).
[21] M.A. Munoz and R. Pastor-Satorras, Phys. Rev. Lett. **90**, 204101 (2003).